



## Reduced Order Model of Wind Turbine Wakes

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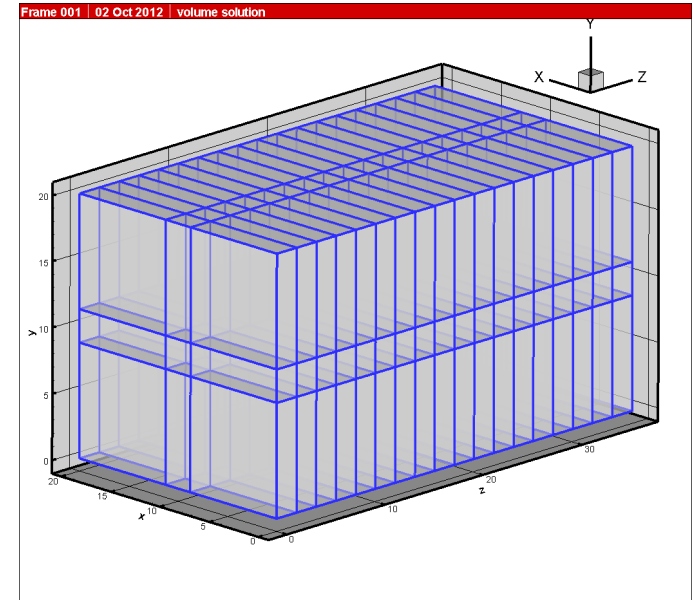
# Reduced Order Model of Wind Turbine Wakes

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Fluid Mechanics, DTU Wind Energy

# Reduced Order Model of Wind Turbine Wakes inside large Wind Farms

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- EllipSys3D: finite volume, multi-block NS solver, parallel, non-dimensional
  - LES
  - Actuator Line method (Sørensen and Shen, 2002)
  - NM80 Turbine with  $R = 40.04\text{m}$
  - Rated  $2.75\text{ MW}$  @  $14\text{ m/s}$
  - Controller
- 
- $20R \times 20R \times 36R \gg 3WT$ , i.e.  $12R$  spacing
  - Uniform Inflow  $\gg 15\text{ m/s}$  in physical space
  - Cyclic BC's on inflow/outflow
  - Free stream BC's (Dirichlet) on vertical and transverse



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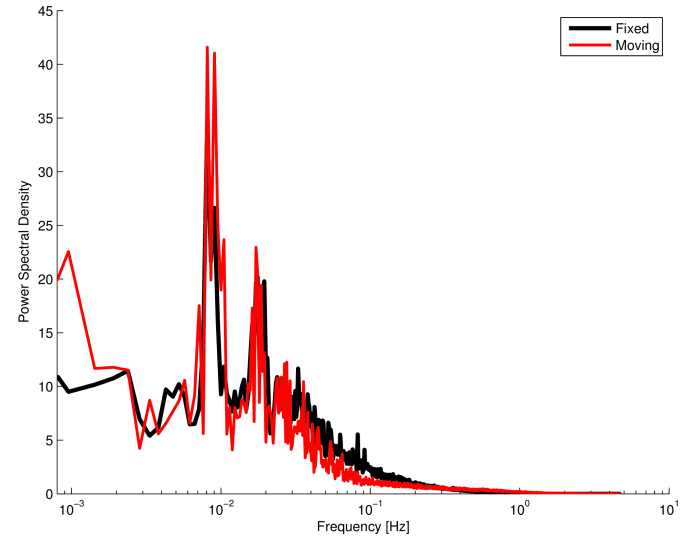
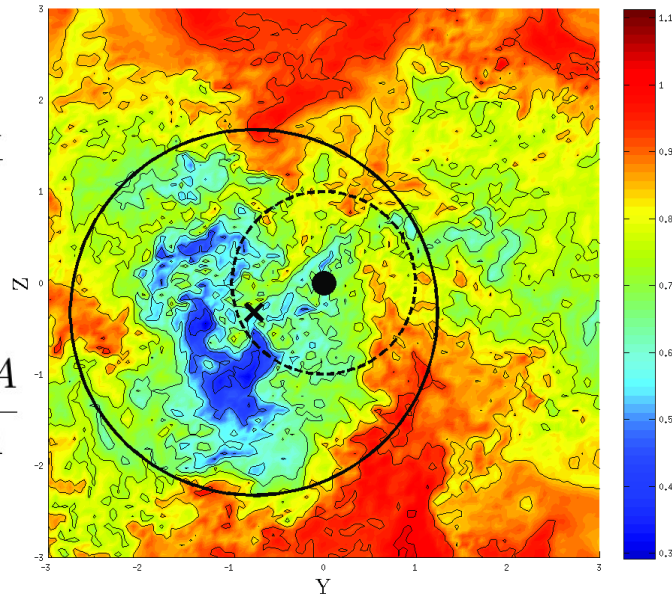
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## Dynamic Wake Meandering

$$y_C = \frac{\int y(1 - \frac{U}{U_0})dA}{\int (1 - \frac{U}{U_0})dA}$$

$$z_C = \frac{\int z(1 - \frac{U}{U_0})dA}{\int (1 - \frac{U}{U_0})dA}$$



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## Reduced Order Model from Proper Orthogonal Decomposition (POD)

Snapshots:

$$\mathbf{U} = [\mathbf{u}'_1 \dots \mathbf{u}'_N] \quad (1)$$

Auto-covariance matrix:

$$\mathbf{R} = \mathbf{U}^T \mathbf{U} \quad (2)$$

Eigenvalue problem of the form:

$$\mathbf{R} \mathbf{G} = \mathbf{G} \mathbf{\Lambda} \quad (3)$$

The spatial POD modes are given as:

$$\phi_k = \frac{\mathbf{U} \mathbf{g}_j}{\|\mathbf{U} \mathbf{g}_j\|} \quad (4)$$

Reconstructing the flow using POD:

$$\mathbf{u}_j \approx \hat{\mathbf{u}}_j = \mathbf{u}_0 + \sum_{k=1}^K \phi_k a_{kj} \quad (5)$$

where  $a_{kj}$  are the elements of  $\mathbf{A} = \mathbf{\Phi}^T \mathbf{U}$ .

Truncating further to construct a Reduced Order Model:

$$\mathbf{u}_j \approx \hat{\mathbf{u}}_j \approx \tilde{\mathbf{u}}_j = \mathbf{u}_0 + \sum_{k=1}^K \phi_k \sum_{m=1}^M (\beta_{k,m}^{(1)} \cos(2\pi f_{k,m} t_j) - \beta_{k,m}^{(2)} \sin(2\pi f_{k,m} t_j)) \quad (6)$$

The coefficients  $\beta_{k,m}$  are determined by optimising the reconstruction in a linear least square sense.

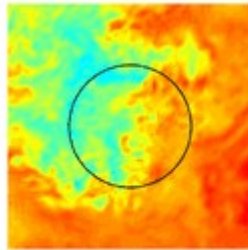
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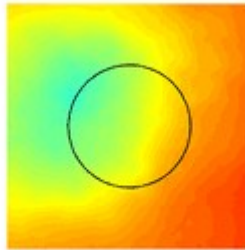


## Streamwise velocity comparison

**CFD**



**POD reconstructed  
4 POD modes**



**Reduced Order Model  
4 POD modes, 9 freq**

